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CONVECTIVE INSTABILITY OF A SYSTEM OF HORIZONTAL LAYERS OF SLIGHTLY COMPRESSIBLE LIQUIDS

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The convective stability of a system of two immiscible liquids with close densities is studied. The densities of the liquids depend nonlinearly on temperature and pressure. It is shown that the state of mechanical equilibrium is unstable. Neutral curves are plotted, and the critical values of the Rayleigh number are found. The calculations are performed for physical parameters characteristic of various northern, central, and southern zones of lake Baikal.

Key words: *instability, interface, neutral curve.*

Introduction. One of the main conditions for deep-water renewal is water temperature and density stratifications. For deep water bodies (of depth more than 200 m), it is necessary to allow for the compressibility of water. A feature of lake Baikal, in addition to its depths (a maximum depth of 1637 m; an average depth of 730 m), is a decrease in the maximum density temperature with increasing depth. This effect is responsible for the mesothermal temperature maximum and is an important factor in the analysis of transfer processes in the lake due to density stratification [1].

According to experimental data [1, 2], the temperature profile of lake Baikal water is nonlinear. In the present study, the coordinate of the salient point of the temperature profile is treated as the coordinate of the interface between two immiscible liquids. In this case, the nonlinear temperature profile is approximated by two linear functions which are exact solutions of the energy equation for each of the liquid regions. This interpretation of the interface taking into account the nonlinear temperature and pressure dependence of the density with small coefficients of thermal expansion and isothermal compressibility allows the liquid layers to be to considered slightly compressible media [3].

1. Formulation of the Problem. We consider gravitational thermal convection in a system of two immiscible liquids with a common interface which are bounded from below by a solid wall and from above by a free surface (Fig. 1). The x and y axes are in the plane of the lower boundary of the layer, and the z axis is directed upward. The point $z = 0$ corresponds to the lower boundary of the layer (solid wall), $z = z_*$ to the interface, and $z = l$ to the free surface. The value of z_* is determined from experimental data [2] as the salient point in the temperature distribution (for months with a mesothermal temperature distribution, z_* is the mesothermal maximum point). The surfaces Γ_t^1 and Γ_t^2 are given by the equations $f_{1,2}(\boldsymbol{x}, t) = 0$, where $\boldsymbol{x} = (x, y, z)$; in particular, for the case considered below, $f_1 = z - z_*$ and $f_2 = z - l$. The density ρ_j is expressed as

$$\rho_j = \rho_0(1 - \beta_j(\theta_j - \theta_{*j})^2). \tag{1}$$

Here ρ_0 is the maximum density reached at the temperature θ_0 called the inversion temperature or the temperature of the thermal-expansion anomaly of the liquid, β_j is the thermal-expansion coefficient, θ_j is the temperature, $\theta_{*j} = \theta_0(1 - \delta_0 p_j)$, p_j is the pressure, and ρ_0, θ_0, and δ_0 are positive constants; the subscript $j = 1$ refers to the lower liquid layer and the subscript $j = 2$ refers to the upper liquid layer. For water, the characteristic value is $\rho_0 = 999.972$ kg/m^3, the inversion temperature is $\theta_0 = 3.98$°C, and $\delta_0 = 5 \cdot 10^{-8}$ Pa^{-1}.

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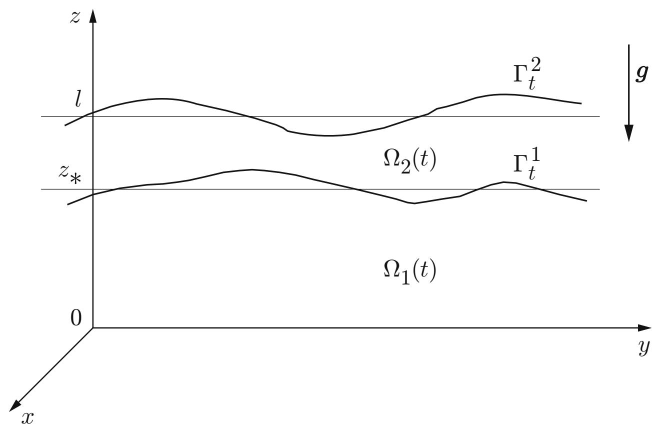

Fig. 1. Flow diagram: Γ_t^1 is the interface and Γ_t^2 is the free surface.

Equation (1) is a simplified version of the equation of state

$$\rho(\theta, p) = \rho_m(p)[1 - \varphi(p)(\theta - \theta_m(p))^2].$$

The form of the functions $\rho_m(p)$, $\varphi(p)$, and $\theta_m(p)$ and a justification of the choice of this equation of state are indicated in [4]. Instead of the functions $\rho_m(p)$ and $\theta_m(p)$, relation (1) contains the zero terms of the Taylor series expansion ρ_0 and θ_0, respectively. The constant δ_0 is determined from the expression for $\theta_m(p)$. For the specified values of the physical parameters (for lake Baikal water), the error in determining the density by Eq. (1) is less than 1%.

In the modeling for lake Baikal, instead of a homogeneous liquid layer we analyze a system of two different liquids with close physical characteristics (temperature, density, etc.) which correspond to experimental data. The approach employed in the present study takes into account the seasonal stability features of Baikal water due to the vertical temperature distribution [1].

For each region Ω_j, the Oberbeck–Boussinesq system of equations is valid:

$$\operatorname{div} \boldsymbol{v}_j = 0, \qquad \frac{\partial \theta_j}{\partial t} + \boldsymbol{v}_j \cdot \nabla \theta_j = \chi_j \Delta \theta_j,$$
$$\rho_0\Big(\frac{\partial \boldsymbol{v}_j}{\partial t} + \boldsymbol{v}_j \nabla \boldsymbol{v}_j\Big) = -\nabla p_j + \mu_j \Delta \boldsymbol{v}_j + \rho_j \boldsymbol{g}. \tag{2}$$

Here $\boldsymbol{v}_j = (u_j, v_j, w_j)$ is the velocity of the jth liquid, χ_j is the thermal diffusivity, μ_j is the viscosity, and $\boldsymbol{g} = (0, 0, -g)$, where g is the acceleration due to gravity.

At the solid wall, the temperature and attachment conditions are imposed:

$$\theta_1 = \Theta_1, \qquad \boldsymbol{v}_1 = 0 \qquad \text{at} \quad z = 0. \tag{3}$$

The boundary conditions are as follows:

— at the interface Γ_t^1,

$$\boldsymbol{v}_1 = \boldsymbol{v}_2, \qquad \theta_1 = \theta_2, \qquad \mathcal{P}_1 \boldsymbol{n} = \mathcal{P}_2 \boldsymbol{n},$$
$$\boldsymbol{v}_1 \cdot \boldsymbol{n} = V_n^1, \qquad k_1 \frac{\partial \theta_1}{\partial \boldsymbol{n}} = k_2 \frac{\partial \theta_2}{\partial \boldsymbol{n}} \qquad \text{at} \quad z = z_*; \tag{4}$$

— at the free surface Γ_t^2,

$$\boldsymbol{v}_2 \cdot \boldsymbol{n} = V_n^2, \qquad \mathcal{P}_2 \cdot \boldsymbol{n} + p_g \cdot \boldsymbol{n} = 0,$$
$$k_2 \frac{\partial \theta_2}{\partial \boldsymbol{n}} + b(\theta_2 - \theta_g) = Q \qquad \text{at} \quad z = l. \tag{5}$$

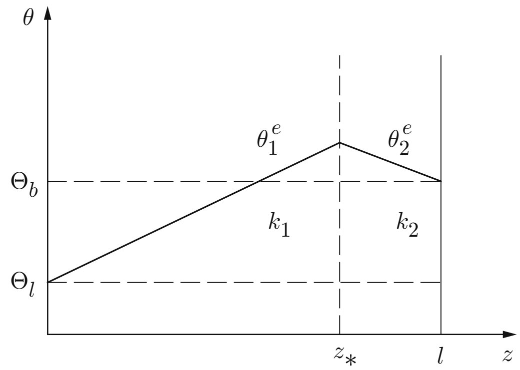

Fig. 2. Distribution of the equilibrium temperature θ_j^e in the liquid layers.

Here $\boldsymbol{n}$ is the normal to the surface Γ_t^j, V_n^j is the velocity Γ_t^j in the normal direction, $\mathcal{P}_j = -p_j + 2\mu_j D_j$ is the stress tensor in the liquid, D_j is the strain rate tensor of the vector field $\boldsymbol{v}_j$, p_g is the gas pressure, k_j is the thermal conductivity of the liquid, b is the interphase heat transfer coefficient, θ_g is the gas temperature, and Q is the specified heat flux through the free surface.

2. Equilibrium State. In the state of mechanical equilibrium, $\boldsymbol{v}_j^e = 0$ and the time derivatives are equal to zero: $\theta_{jt}^e = p_{jt}^e = 0$. The energy equation implies that θ_j^e are linear functions z of the form

$$\theta_j^e(z) = A_j z + B_j, \tag{6}$$

where the constants A_1 and B_1 are determined from the boundary conditions at the interface and the solid wall, respectively:

$$A_1 = k_2 A_2 / k_1, \qquad B_1 = \Theta_1,$$

and the constants A_2 and B_2 are found from the conditions at the free surface and the interface, respectively:

$$A_2 = \frac{Q - bB_2 + b\theta_g}{k_2 + bl}, \qquad B_2 = \frac{\Theta_1(k_2 + bl) + (Q + b\theta_g) z_* (k_2/k_1 - 1)}{k_2 + bl + bz_*(k_2/k_1 - 1)}.$$

This temperature distribution in the layers (Fig. 2) is in good agreement with data of full-scale observations at lake Baikal [2, 5]. (In Fig. 2, Θ_b is the free-surface temperature.) The complex temperature profile is approximated by two straight lines $\theta_1^e(z)$ and $\theta_2^e(z)$ in the regions Ω_1 and Ω_2, respectively.

From the momentum equation, we determine the pressure

$$p_1^e = \frac{1}{\sqrt{|C_1|}} \frac{C_3 H(z) - 1}{C_3 H(z) + 1} - Dz - E - \rho_0 g z,$$

where

$$C_1 = \frac{C}{D}, \quad C = \rho_0 g \alpha \theta_0^2 \delta_0^2, \quad D = \frac{A_1 - \theta_0 \delta_0 \rho_0 g}{\theta_0 \delta_0}, \quad H(z) = \exp\,(2\sqrt{|C_1|}\,(Dz + E)),$$

$$E = \frac{B_1 - \theta_0}{\theta_0 \delta_0}, \quad C_3 = \frac{1 + \sqrt{|C_1|}\,G}{H(z_*)(1 - \sqrt{|C_1|}\,G)}, \quad G = p_2(z_*) + Dz_* + E + \rho_0 g z_*,$$

and pressure

$$p_2^e = \frac{1}{\sqrt{|C_4|}} \frac{C_5 H_2(z) - 1}{C_5 H_2(z) + 1} - D_2 z - E_2 - \rho_0 g z,$$

where

$$C_4 = \frac{C_2}{D_2}, \quad C_2 = \rho_0 g \alpha \theta_0^2 \delta_0^2, \quad D_2 = \frac{A_2 - \theta_0 \delta_0 \rho_0 g}{\theta_0 \delta_0}, \quad H_2(z) = \exp\,(2\sqrt{|C_4|}\,(D_2 z + E_2)),$$

$$E_2 = \frac{B_2 - \theta_0}{\theta_0 \delta_0}, \quad C_5 = \frac{1 + \sqrt{|C_4|}\,G_2}{H_2(l)(1 - \sqrt{|C_4|}\,G_2)}, \quad G_2 = p_g + D_2 l + E_2 + \rho_0 g l.$$

The profiles of p_j^e are convex downward and are nearly linear.

Thus, we obtained a steady-state solution p_j^e, θ_j^e of the boundary-value problems (2)–(5) that corresponds to the state of mechanical equilibrium $\boldsymbol{v}_j^e = 0$.

3. Problem of Small Perturbations of Equilibrium. We formulate the problem of the stability of mechanical equilibrium against small perturbations. For this, we introduce the determining dimensionless parameters. The width l_* of the lower layer will be used as the characteristic length scale, the difference $\Theta = \theta_1 - \theta_0$ as the temperature scale, and the velocity of convective rise of a heated fluid particle $v_* = \sqrt{gl_*\beta\Theta^2}$ as the velocity scale. For the density and pressure, we will use the scales ρ_0 and $\rho_0 v_*^2$, respectively. The temperature will be reckoned from the temperature of the lower boundary Θ_1, and the pressure from the hydrostatic pressure.

We introduce dimensionless variables $\boldsymbol{\xi} = (\xi, \eta, \zeta)$, τ such that

$$\boldsymbol{x} = (x, y, z) = \boldsymbol{\xi} l_*, \qquad t = \frac{l_*}{v_*}\tau, \qquad l_* = \frac{l}{\lambda}, \qquad \lambda = \frac{\Theta_1 - \Theta_b}{\Theta},$$

$$p_j = \rho_0 v_*^2 p_j', \qquad \boldsymbol{v}_j = v_* \boldsymbol{v}_j', \qquad \theta_j = \Theta\theta_j'.$$

Here λ is the inversion parameter, Θ_b is the free-surface temperature calculated by formula (6) with constant A_2 and B_2, and p_j', v_j', and θ_j' are dimensionless functions of the pressure, velocity, and temperature, respectively.

As the scales for the coefficients ν, χ, and β and the equilibrium temperature gradients A we use their mean arithmetic values

$$\nu_* = \frac{\nu_1 + \nu_2}{2}, \qquad \chi_* = \frac{\chi_1 + \chi_2}{2}, \qquad \beta_* = \frac{\beta_1 + \beta_2}{2}, \qquad A_* = \frac{A_1 + A_2}{2}.$$

Problem (2)–(5) is determined by the following dimensionless parameters: $\varepsilon = \beta\Theta^2$, $\varepsilon_T = (\theta_0\delta_0\rho_0 v_*^2)/\Theta$, $\mathrm{R} = 2/(\mu_*\delta_*)$ is the Rayleigh number, $\mu_* = \nu_*/(l_* v_*)$ is the kinematic viscosity parameter (the reciprocal of the Reynolds number), and $\delta_* = \chi_*/(l_* v_*)$ is the Fourier number.

Let $\boldsymbol{v}_{dj}(\boldsymbol{\xi}, \tau) = \boldsymbol{v}_j(\boldsymbol{\xi}, \tau) + \delta_* \boldsymbol{V}_j(\boldsymbol{\xi}, \tau)$, $p_{dj}(\boldsymbol{\xi}, \tau) = p_j(\boldsymbol{\xi}, \tau) + \mu_*\delta_* P_j(\boldsymbol{\xi}, \tau)$, and $\theta_{dj}(\boldsymbol{\xi}, \tau) = \theta_j(\boldsymbol{\xi}, \tau) + T_j(\boldsymbol{\xi}, \tau)$, where $\boldsymbol{V}_j = (U_j, V_j, W_j)$, P_j and T_j are the perturbations, and $\boldsymbol{v}_j, p_j, \theta_j$ is the main solution. The form of the functions $\boldsymbol{v}_d$, p_d, and θ_d describing the perturbed motion is chosen to simplify the subsequent transformations. Linearizing the total problem, for the velocity, temperature, and pressure perturbations in each of the liquids, we obtain the following boundary-value problem:

$$\begin{gathered} U_{j\xi} + V_{j\eta} + W_{j\zeta} = 0, \qquad T_{j\tau} + \delta_* h_1 W_j = \delta_* \Delta T_j, \\ U_{j\tau}/\mu_* = -P_{j\xi} + \Delta U_j, \qquad V_{j\tau}/\mu_* = -P_{j\eta} + \Delta V_j, \\ W_{j\tau}/\mu_* = -P_{j\zeta} + \Delta W_j + \mathrm{R}(\theta_j - \gamma + \varepsilon_T p_j)T_j + 2(\theta_j - \gamma + \varepsilon_T p_j)\varepsilon_T P_j \end{gathered} \tag{7}$$

($h_1 = A_* l_*/\Theta$ and $\gamma = \theta_0/\Theta$).

The boundary conditions are as follows:

— at the solid wall,

$$\zeta = 0: \qquad U_1 = V_1 = W_1 = 0, \qquad T_1 = 0; \tag{8}$$

— at the interface,

$$\begin{gathered} \zeta = 1: \quad U_1 = U_2, \quad V_1 = V_2, \quad W_1 = W_2, \quad T_1 = T_2, \quad T_{1\zeta} = kT_{2\zeta}, \\ U_{1\zeta} + W_{1\xi} = U_{2\zeta} + W_{2\xi}, \qquad V_{1\zeta} + W_{1\eta} = V_{2\zeta} + W_{2\eta} = 0, \\ P_1 - P_2 + 2(\rho_2 - \rho_1)(\nu_2 - \nu_1)(W_{2\zeta} - W_{1\zeta}) = [p_{2\zeta} - p_{1\zeta} - (\rho_2 - \rho_1)/\varepsilon]R_1\mathrm{R}/2. \end{gathered} \tag{9}$$

Here $k = k_2/k_1$, ρ_j, and ν_j are the dimensionless relative values of the thermal conductivity, density, and kinematic viscosity, respectively, and $R_1 = R_1(\xi, \eta, \tau)$ is the local deviation of the interface from its unperturbed state along the normal.

The conditions on the free boundary are written as

$$\begin{gathered} \zeta = \lambda: \qquad -R_{2\tau} + \delta_* W_2 = 0, \qquad U_{2\zeta} + W_{2\xi} = 0, \qquad V_{2\zeta} + W_{2\eta} = 0, \\ -\mu_*\delta_* P_2 + 2\mu_*\delta_* W_{2\zeta} = h_2 R_2, \qquad \Theta_{2\zeta} + \mathrm{Bi}(T_2 + h_1 R_2) = 0, \end{gathered} \tag{10}$$

where $h_2 = \partial p_2/\partial\zeta$, $R_2 = R_2(\xi, \eta, \tau)$ is the perturbation of the free boundary, and $\mathrm{Bi} = bl_*/k_2$ is the Biot number.

Let us consider normal perturbations proportional to $\exp[i(\alpha_1\xi + \alpha_2\eta - Ct)]$, where $C = C_r + iC_i$ is the complex decrement and α_1 and α_2 are the wavenumbers along the x and y axes, respectively. For the amplitudes of the normal perturbations, we obtain a spectral boundary-value problem to which the Squire transformation $Z_j = i\alpha_1 U_j + i\alpha_2 V_j$ applies. After the transformation, system (7) is written as

$$Z_j + W_j' = 0, \qquad -iCT_j + \delta_* h_1 W_j = \delta_*(T_j'' - \alpha^2 T_j),$$

$$-iCZ_j/\mu_* = \alpha^2 P_j + Z_j'' - \alpha^2 Z_j, \tag{11}$$

$$-iCW_j/\mu_* = -P_j' + W_j'' - \alpha^2 W_j + \mathrm{R}(\theta_j - \gamma + \varepsilon_T p_j)T_j + 2\varepsilon_T(\theta_j - \gamma + \varepsilon_T p_j)P_j$$

($\alpha^2 = \alpha_1^2 + \alpha_2^2$ is a modified wavenumber).

Boundary conditions (8)–(10) become

$$\zeta = 0\colon \qquad Z_1 = 0, \qquad W_1 = 0, \qquad T_1 = 0,$$

$$\zeta = 1\colon \qquad Z_1 = Z_2, \qquad W_1 = W_2, \qquad T_1 = T_2, \qquad R_1 = iW_1/C,$$

$$P_1 - P_2 + 2(\rho_2 - \rho_1)(\nu_2 - \nu_1)(W_2' - W_1') = [p_2' - p_1' - (\rho_2 - \rho_1)/\varepsilon]R_1\mathrm{R}/2,$$

$$Z_2' - \alpha^2 W_2 = Z_1' - \alpha^2 W_1, \qquad T_2' = kT_1', \tag{12}$$

$$\zeta = \lambda\colon \qquad -P_2 + 2W_2' = \frac{\mathrm{R}}{2}\, h_2\, \frac{i\delta_*}{C}\, W_2, \quad Z_2' - \alpha^2 W_2 = 0, \quad T_2' + \mathrm{Bi}\Big(T_2 + h_1\, \frac{i\delta_*}{C}\, W_2\Big) = 0.$$

The boundary-value problem (11), (12) is an eigenvalue problem for the complex decrement C. In order that the equilibrium state p_j^e, θ_j^e be stable against small perturbations of the specified form, it is necessary and sufficient that the imaginary part C of all eigenvalues C_i be negative.

4. Long-Wave Asymptotics. The unknown functions Z_j, W_j, P_j, T_j, and C are represented as follows (as $\alpha \to 0$):

$$(Z_j, W_j, P_j, T_j, C) = (Z_{j0}, W_{j0}, P_{j0}, T_{j0}, C_0) + \alpha(Z_{j1}, W_{j1}, P_{j1}, T_{j1}, C_1) + \ldots\ .$$

Substituting the indicated expansion into system (11), we write the obtained equations in the zero approximation

$$Z_{j0}'' = -iC_0 Z_{j0}/\mu_* \tag{13}$$

with the boundary conditions

$$Z_{10} = 0 \quad \text{at} \quad \zeta = 0, \qquad Z_{20}' = 0 \quad \text{at} \quad \zeta = \lambda,$$
$$Z_{10} = Z_{20}, \quad Z_{10}' = Z_{20}' \quad \text{at} \quad \zeta = 1. \tag{14}$$

Multiplying each of Eqs. (13) into the complex conjugate quantity Z_{j0}^*, integrating over the segment $[0, 1]$ for $j = 1$ and over the segment $[1, \lambda]$ for $j = 2$, and summing the resulting equations, we have

$$\int_0^1 |Z_{10}'|^2\, d\zeta + \int_1^\lambda |Z_{20}'|^2\, d\zeta = \frac{iC_0}{\mu_*}\Big(\int_0^1 |Z_{10}|^2\, d\zeta + \int_1^\lambda |Z_{20}|^2\, d\zeta\Big).$$

From this it follows that $iC_0/\mu_* > 0$. Because $\mu_* > 0$, it follows that $iC_0 > 0$. Therefore, $C_0 = iC_i$ is a purely imaginary number and $C_i < 0$. This implies that long-wave perturbations damp monotonically.

Let us specify the form of C_0. We denote $iC_0/\mu_* = \mu$. Then, Eq. (13) can be written as

$$Z_{j0}'' + \mu Z_{j0} = 0.$$

Because $\mu > 0$, it follows that $Z_{j0} = c_{j1}\cos\sqrt{\mu}\,\zeta + c_{j2}\sin\sqrt{\mu}\,\zeta$. In the last expression, the constants c_{j1} and c_{j2} are determined from boundary conditions (14). In this case, $c_{11} = 0$, $\mu = (\pi n + \pi/2)^2/\lambda^2$ (n is a natural number), and

$$C_0 = -i\mu_*(\pi n + \pi/2)/\lambda^2. \tag{15}$$

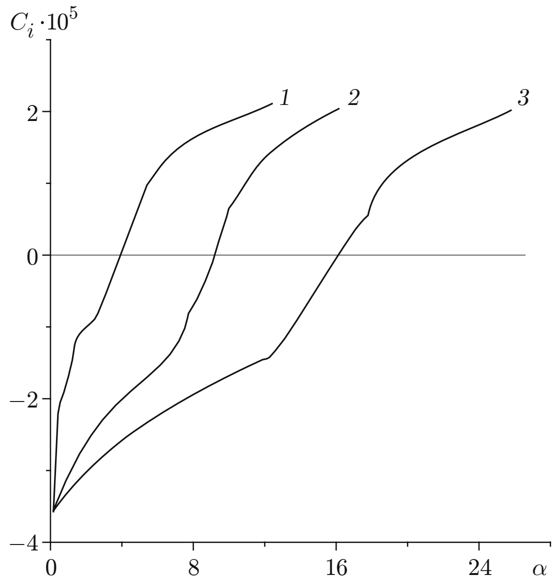

Fig. 3

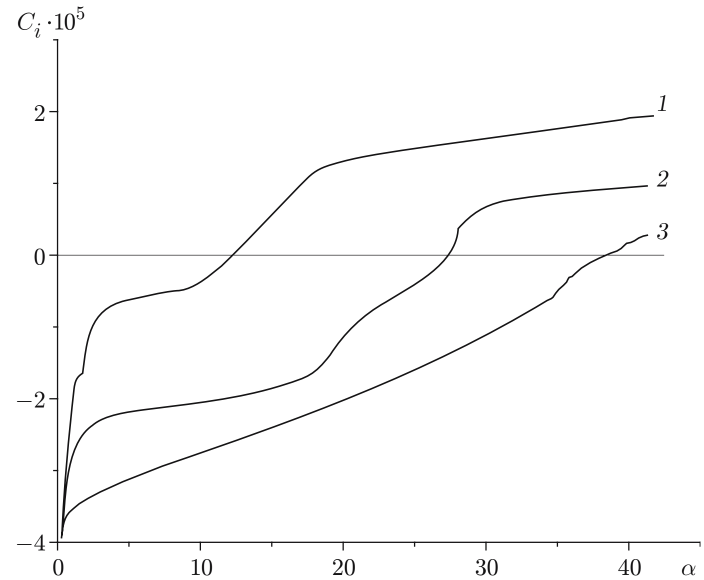

Fig. 4

Fig. 3. Complex decrements $C_i(\alpha)$ calculated for medium depths: 1) northern zone of lake Baikal ($\alpha_* = 3.91$, $l_* = 339$ m, and Bi = 0.54); 2) southern zone ($\alpha_* = 9.01$, $l_* = 610$ m, and Bi = 0.76); 3) central zone ($\alpha_* = 16.6$, $l_* = 553$ m, and Bi = 0.98).

Fig. 4. Complex decrements $C_i(\alpha)$ calculated for the maximum depths: 1) northern zone of lake Baikal ($\alpha_* = 12.3$, $l_* = 765$ m, and Bi = 1.23); 2) southern zone ($\alpha_* = 26.94$, $l_* = 1243$ m, and Bi = 1.54); 3) central zone ($\alpha_* = 38.61$, $l_* = 1387$ m, and Bi = 2.47).

5. Numerical Solution. The spectral problem (11), (12) is solved by an orthogonalization method [6]. To find the eigenvalue C, it is necessary to know the initial approximation C_0, which is chosen from condition (15).

We analyzed the stability of a system of horizontal layers of slightly compressible liquids with a common interface for the following parameter values: $\theta_g = 287$ K, $p_g = 101{,}300$ Pa, $\nu_* = 1.57 \cdot 10^{-6}$ m^2/sec, $\chi_* = 1.323 \cdot 10^{-7}$ m^2/sec, $\beta_1 = 8.41 \cdot 10^{-6}$ K^{-2}, $\beta_2 = 8.73 \cdot 10^{-6}$ K^{-2}, $k_1 = 0.556$ W/(m$\cdot$K), and $k_2 = 0.562$ W/(m$\cdot$K). These values correspond to the parameter values for lake Baikal water. For the indicated values of the physical parameters, we obtained the dependence of $C_i = \operatorname{Im} C$ on the wavenumber α.

The calculations were performed for the medium and maximum depths l of the southern, central, and northern zones of lake Baikal. The heat fluxes Q characteristic of these zones were taken into account. Figures 3 and 4 show curves of $C_i(\alpha)$ for the medium and maximum depths, respectively, in the southern, central, and northern zones of lake Baikal (α_* are the critical wavenumbers). The medium depth l is 810 m in the southern zone, 803 m in the central zone, and 564 m in the northern zone. The maximum depth is 1443 m in the southern zone of lake Baikal, 1637 m in the central zone, and 990 m in the northern zone. The obtained values of α_* correspond to the following dimensional values of the critical wavelength $\lambda = 2\pi/\alpha_*$: for the northern zone of lake Baikal, $\lambda_1 = 544.5$ m for the medium depths and $\lambda_2 = 390.6$ m for the maximum depths: for the central zone, $\lambda_1 = 209.01$ m and $\lambda_2 = 225.6$ m, respectively, and for the southern zone, $\lambda_1 = 425.2$ m and $\lambda_2 = 289.76$ m, respectively. An analysis of the results suggests that the heat transfer has a stabilizing effect on the stability of equilibrium.

Calculations were also conducted for the case of identical thermal-expansion coefficients β_1 and β_2. The difference between the results obtained for this case and the results obtained in the present work for $\beta_1 \neq \beta_2$ is about 10^{-11} for values of $C_i(\alpha)$ and about 10^{-2} for values of $\mathrm{R}(\alpha)$.

The stability boundary is determined from the relation $C_i(\mathrm{R}) = 0$. Neutral perturbations correspond to the case $C_i = 0$. Setting $C = 0$ in problem (11), (12), we obtain the neutral stability curves. In the calculations, the

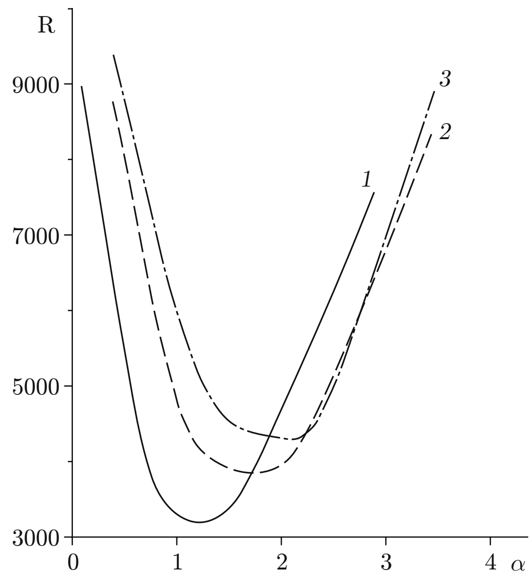

Fig. 5. Neutral curves of R(α): 1) Bi = 0.2, $R_* = 3111.24$, and $\alpha_* = 1.21$; 2) Bi = 1, $R_* = 3771.2$, and $\alpha_* = 1.8$; 3) Bi = 2, $R_* = 4266.17$, and $\alpha_* = 2.1$.

values of the Biot number was varied and the value of l was set equal to 1000 m in all cases. Figure 5 gives a curve of the Rayleigh number versus the wavenumber (neutral curves). For different values of the Biot number in the figure, we give the critical Rayleigh numbers R_* that are the minimum values on the corresponding neutral curves and the critical wavenumbers α_* for which the quantities R_* are reached. It is evident that as the Biot number decreases, the critical Rayleigh numbers decrease and the region of instability is shifted toward larger wavenumbers.

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